PMM U.S.S.R.,Vol.45,pp. 685-689
0021-8928/82/5 0685 $7.50/0
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UDC 539.3

PROPERTIES OF SOLUTIONS OF THE DYNAMIC PROBLEMS OF THE GENERALIZED COUPLED THERMOELASTICITY*

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Analytic properties of solutions of fundamental dynamic problems of the generalized coupled thermoelasticity with the finite rate of heat propagation taken into account, are investigated. In the particular case of infinite rate of heat propagation the results obtained sharpen the results and conclusions given in /1,2/. A problem of thermal shock at the surface of a spherical cavity is studied.

1. Fundamental solutions of generalized thermoelasticity. Dynamic processes taking place in thermoelastic media can be described by the following system of equations /3/:

$$\begin{aligned}
&A(\partial_x)u-\gamma\,\mathrm{grad}\,u_4-\rho\frac{\partial^2 u}{\partial t^2}=-\rho F,\quad \gamma=\alpha_\theta(3\lambda+2\mu)\\
&\Delta u_4-\frac{1}{\varkappa}l\frac{\partial u_4}{\partial t}-\eta l\frac{\partial}{\partial t}\,\mathrm{div}\,u=-\frac{l}{\varkappa}Q,\quad l=1+t_r\frac{\partial}{\partial t}\\
&A(\partial_x)=\|A_{kj}(\partial_x)\|_{3\times 3},\quad A_{kj}(\partial_x)=\delta_{kj}\mu\Delta+(\lambda+\mu)\frac{\partial^2}{\partial x_k\partial x_j},\\
&k,j=1,2,3.
\end{aligned}\tag{1.1}$$

Here $u=(u_1,u_2,u_3)$ is the displacement vector, u_4 is the temperature, $F=(F_1,F_2,F_3)$ is the mass force density vector, $Q(x,t)$ is the specific intensity of the heat sources, α_θ is the linear thermal expansion coefficient, λ and μ are the Lamé coefficients, t_r is the heat flux relaxation time, η is the coupling coefficient, $\varkappa$ is the heat diffusion coefficient and Δ is the Laplace operator. In addition to (1.1), we shall consider the corresponding elliptical system for the Fourier transforms with respect to time of the functions sought. The system describes the thermoelastic pseudooscillations at $\mathrm{Im}\,\omega>0$ and steady thermoelastic oscillations at $\mathrm{Im}\,\omega=0$:

$$\begin{aligned}
&B(\partial_x,\omega)U^F=H^F\\
&B(\partial_x,\omega)=\|B_{kj}(\partial_x,\omega)\|_{4\times 4}\\
&B_{kj}(\partial_x,\omega)=A_{kj}(\partial_x)+\rho\omega^2,\quad B_{k4}(\partial_x,\omega)=-\gamma\frac{\partial}{\partial x_k}\\
&B_{4j}(\partial_x,\omega)=\eta\Omega\frac{\partial}{\partial x_j},\quad B_{44}(\partial_x,\omega)=\Delta+\Omega\varkappa^{-1},\quad \Omega=i\omega(1-i\omega t_r)\\
&H^F=(H_1^F,H_2^F,H_3^F,H_4^F),\ H_k^F=-\rho F_k^F,\ H_4^F=-\varkappa^{-1}(1-i\omega t_r)Q^F,\ k,j=1,2,3
\end{aligned}\tag{1.2}$$

Here U^F denotes the Fourier transform, with respect to time, of the four-component vector $U=(u_1,u_2,u_3,u_4)$

$$U^F(x,\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}U(x,t)e^{i\omega t}\,dt\tag{1.3}$$

and H^F is the four-component vector Fourier transform of the vector of mass forces and heat sources.

The matrix of fundamental solutions of the homogeneous equation (1.2) has the form

$$\begin{aligned}
&T(x,\omega)=\|T_{kj}(x,\omega)\|_{4\times 4}\\
&T_{kj}(x,\omega)=\sum_{p=1}^{3}\Big\{(1-\delta_{k4})(1-\delta_{j4})\Big(\frac{\delta_{kj}}{2\pi\mu}\delta_{3p}-\alpha_p\frac{\partial^2}{\partial x_k\partial x_j}\Big)+\\
&\beta_p\Big[\eta\Omega\delta_{k4}(1-\delta_{j4})\frac{\partial}{\partial x_j}-\gamma\delta_{j4}(1-\delta_{k4})\frac{\partial}{\partial x_k}\Big]+\delta_{k4}\delta_{j4}\gamma_p\Big\}\frac{\exp(i\lambda_p|x|)}{|x|}
\end{aligned}\tag{1.4}$$

*Prikl.Matem.Mekhan.,45,No.5,912-918,1981

$$\alpha_p = (1 - \varkappa^{-1}\Omega\lambda_p^{-2})\,\beta_p - \frac{\delta_{3p}}{2\pi\rho\omega^2}, \quad \beta_p = \frac{(-1)^p(\delta_{1p} + \delta_{2p})}{2\pi(\lambda + 2\mu)(\lambda_2^2 - \lambda_1^2)}$$
$$\gamma_p = (\lambda_p^2 - k_1^2)(\lambda + 2\mu)\,\beta_p, \quad k_1^2 = \rho\omega^2(\lambda + 2\mu)^{-1}$$
$$\sum_{p=1}^{3}\alpha_p = 0, \quad \sum_{p=1}^{3}\beta_p = 0, \quad 2\pi\sum_{p=1}^{3}\gamma_p = 1$$
$$\lambda_1^2 + \lambda_2^2 = \Omega\varkappa^{-1} + \gamma\eta\Omega(\lambda + 2\mu)^{-1} + k_1^2, \quad \lambda_1^2\lambda_2^2 = \Omega\varkappa^{-1}k_1^2$$
$$\lambda_3^2 = \rho\omega^2\mu^{-1}.$$

We note that, as $t_r \to 0$ (1.4) yields a representation for the matrix of the fundamental solutions of the corresponding system of equations of thermoelasticity, for the case of infinite rate of propagation of thermal perturbations obtained in /1,4/. Using (1.4), we write the characteristic thermoelastic parameters in the form (c_q is rate of heat propagation):

$$\lambda_k = \frac{c_1}{\varkappa}\sqrt{\frac{\chi}{2}}\,[a\chi + i(1+\varepsilon) \pm \sqrt{p}E]^{1/2}, \quad k = 1, 2 \tag{1.5}$$
$$E = [(\chi + i\chi_2^0 - \chi_1^0)(\chi + i\chi_2^0 + \chi_1^0)]^{1/2}$$
$$\chi = \frac{\omega}{\omega^*}, \quad \omega^* = \frac{c_1^2}{\varkappa}, \quad c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad M = \frac{c_1}{c_q}$$
$$c_q^2 = \frac{\varkappa}{t_r}, \quad \varepsilon = \frac{\varkappa\gamma\eta}{\lambda + 2\mu}, \quad \chi_1^0 = \frac{2\sqrt{\varepsilon}}{p}, \quad \chi_2^0 = \frac{q}{p}$$
$$p = a^2 - 4M^2, \; q = a(1+\varepsilon) - 2, \; a = 1 + M^2(1+\varepsilon)$$

2. Basic properties of the solutions of dynamic thermoelasticity problems.

Lemma 1. The characteristic parameters λ_k have the following properties:

a) when $M^2 < (1-\varepsilon)(1+\varepsilon)^{-2}$, the functions $\lambda_k = \lambda_k(\omega), k = 1, 2$ have second order branching points in the half-plane $\mathrm{Im}\,\omega = \sigma > 0$

$$\chi_\pm = \pm\chi_1^0 - i\chi_2^0 \tag{2.1}$$

b) $\lambda_k(\omega) = O(|\omega|)$ as $|\omega| \to \infty$, $k = 1, 2, 3$;

c) $\mathrm{Im}\,\lambda_k > 0$ in the half-plane $\mathrm{Im}\omega > 0$, $k = 1, 2, 3$;

d) $\mathrm{Re}\,\lambda_k \gtrless 0$ when $\mathrm{Re}\,\omega \gtrless 0$, $k = 1, 2, 3$.

Proof. The properties a) and b) follow directly from the expression (1.5) for the characteristic parameters. The properties c) and d) are obvious for $k = 3$. To prove the properties c) for $k = 1, 2$ we consider the function E of complex variable χ. We determine E unambiguously by considering the complex plane $\chi = \chi_1 + i\chi_2$ as a two-sheeted surface where the sheets are joined together along the edges of the cuts shown in the Fig.1. Then, fixing the branch of the root by the condition that $\mathrm{Im}\,E > 0$ when $\chi_1 = 0$, we obtain

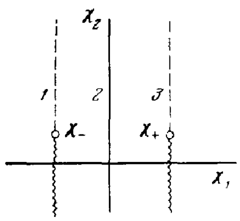

Fig.1

$$\mathrm{Re}\,E = \begin{cases} -L_2N_1 - L_1N_2 & \text{in region 1} \\ L_2N_1 - L_1N_2 & \text{in region 2} \\ L_2N_1 + L_1N_2 & \text{in region 3} \end{cases} \tag{2.2}$$

$$\mathrm{Im}\,E = \begin{cases} L_1N_1 - L_2N_2 & \text{in regions 1 and 3} \\ L_1N_1 + L_2N_2 & \text{in region 2} \end{cases} \tag{2.3}$$

where

$$L_{1,2} = \left[\frac{R_2 \pm (\chi_2 + \chi_2^0)}{2}\right]^{1/2}, \qquad N_{1,2} = \left[\frac{R_1 \pm (\chi_2 + \chi_2^0)}{2}\right]^{1/2}$$
$$R_{1,2} = [(\chi_2 + \chi_2^0)^2 + (\chi_1 \mp \chi_1^0)^2]^{1/2}$$

Fixing the branches of the functions $\lambda_m(\omega)$ $(m = 1, 2)$ on the upper sheets of the four-sheeted Riemannian surfaces by means of the conditions that $\mathrm{Im}\lambda_m > 0$ when $\chi_2 \to +\infty$, we obtain from (2.2) and (2.3)

$$\mathrm{Im}\,\lambda_m = \frac{c_1}{\sqrt{8\varkappa}}\left(\sqrt{Z_m + X_m}\sqrt{k_2 + \chi_2} - \sqrt{Z_m - X_m}\sqrt{k_2 - \chi_2}\right) \tag{2.4}$$
$$Z_m = \sqrt{X_m^2 + Y_m^2}, \qquad X_m = a\chi_2 + 1 + \varepsilon - (-1)^m\sqrt{p}\,\mathrm{Im}\,E$$
$$Y_m = a\chi_1 - (-1)^m\sqrt{p}\,\mathrm{Re}\,E, \qquad k_2 = \sqrt{\chi_1^2 + \chi_2^2}$$

According to (2.4), the condition $\operatorname{Im}\lambda_m > 0$ $(m = 1, 2)$ is equivalent, for $\chi_2 > 0$, to the inequality

$$X_m > 0 \tag{2.5}$$

which is obvious for $m = 1$ in region 2 and in parts of the regions 1 and 3 for $\chi_2 > \max\{-\chi_2^0, 0\}$, since here $\operatorname{Im} E > 0$ in accordance with (2.2). The inequality (2.5) is also obvious for $m = 2$ in parts of the regions 1 and 3 for $0 < \chi_2 < \max\{-\chi_2^0, 0\}$, since here we have $\operatorname{Im} E < 0$ in accordance with (2.2). In the remaining parts of the regions 1,2 and 3 where the inequality (2.5) is not obvious, it can be reduced to the form

$$[\chi_2^2 (p + 4M^2) + 2\chi_2 (1 + \varepsilon) a + (1 + \varepsilon)^2 + p\chi_1^2]\, \chi_2 (1 + M^2\chi_2) + \varepsilon\chi_1^2 > 0 \tag{2.6}$$

which is obvious, and this proves the property c). Property d) is proved in the same manner. We note that the property c) was proved for a particular case of $M = 0$ in /1/ under the constraint $\operatorname{Im}\chi > -\chi_2^0$, i.e. for $\operatorname{Im}\omega > (\lambda + 2\mu)(1 - \varepsilon)(\rho\varkappa)^{-1}$.

Theorem 1. The elements of the matrix of fundamental solutions $T_{kj}(x, \omega)$ of the system of homogeneous equations (1.2) are analytic functions of the complex variable ω in the halfplane $\operatorname{Im}\omega > 0$.

Proof. From (1.4) and property a) of Lemma 1 it follows that the branch points (2.1) may represent the only singularities of the elements of the matrix of fundamental solutions in the half-plane $\operatorname{Im}\omega > 0$. It can be shown that the elements of the matrix (1.4) can be written in the form

$$T_{kj} = A_{kj}\frac{e^{i\lambda_1|x|} - e^{i\lambda_2|x|}}{\lambda_1^2 - \lambda_2^2} + B_{kj}(e^{i\lambda_1|x|} + e^{i\lambda_2|x|}) \tag{2.7}$$

where A_{kj} and B_{kj} are analytic functions of the parameter ω at $\operatorname{Im}\omega > 0$. From (2.7), expanding the radicals λ_1 and λ_2 near the branch points (2.1) into the generalized power series, we obtain

$$\begin{aligned} T_{kj} &= A_{kj}^* E_\pm^{-1} \sin(E_\pm C\,|x|) + B_{kj}^* \cos(E_\pm C\,|x|) \\ E_\pm &= (\chi \pm \chi_1^0 + i\chi_2^0)^{1/2} \end{aligned} \tag{2.8}$$

Here A_{kj}^*, B_{kj}^* and C are analytic functions of the variable ω in the neighborhood of the points of expansion. Expressing the elements of the matrix of fundamental solutions near the points $\chi_\pm$ in the form (2.8), proves the theorem.

Transferring now the classification of the fundamental boundary value problems of thermoelasticity I±, II±, III±, IV± /1/ to the generalized thermoelasticity, we formulate the following theorem.

Theorem 2. If $\operatorname{Im}\omega > 0$, then the problems corresponding to I±, II±, III±, IV± are solvable for the Fourier transforms (pseudooscillations), their solutions are unique and can be written in the form of the corresponding thermoelastic potentials of the corresponding problems of /1/.

The assertion follows from the property c) of the characteristic parameters after carrying out the manipulations analogous to those given in Ch.X of /1/. We shall assume now that the initial parameters of the problem, i.e. its functions describing the behavior of components of the deformation and temperature fields and of their necessary derivatives at the region boundary, as well as the mass force and heat source densities, can increase exponentially with time. Then we have the following theorem.

Theorem 3. If $\sigma > 0$ is the largest index of exponential growth with respect to time of the initial parameters of the corresponding boundary value problems I±, II±, III±, IV±, then the Fourier transforms of their solutions are analytic functions of the variable ω when $\operatorname{Im}\omega \geqslant \sigma' > \sigma > 0$. The proof follows directly from Theorems 1 and 2 exactly as in /1/.

Corollary. If the exponential growth index with respect to time of the initial parameters of the corresponding problems is zero, then the Fourier transforms of their solutions are analytic functions when $\operatorname{Im}\omega \geqslant \sigma' > 0$.

The properties of the Fourier transforms of solutions of the corresponding problems make it possible to write the solutions themselves in the form

$$U(x, t) = \frac{1}{\sqrt{2\pi}} \int_\Gamma U^F(x, \omega)\, e^{-i\omega t}\, d\omega$$

where the contour Γ lies in the upper half-plane and is chosen in accordance with the causality principle /5,6/ so that is passes all singularities of the transforms from above. In an important particular case when the exponential growth index with respect to time of the initial parameters of the problems $\sigma = 0$, the contour Γ coincides with the real axis, passing all possible singularities lying on this axis through the upper half-plane of the complex variable ω.

The results obtained imply that the thermoelastic medium cannot be active (unstable, becoming stronger) /6/, i.e. the perturbations within it cannot continue to grow in time after the source of perturbations has become inactive. Neither can they grow more rapidly than the perturbations of the source, and this satisfies the analytic criterion of the causality principle /6/. This corollary rectifies the results of /7/. The results obtained follow from Theorem 3, which generalizes, and at the same time sharpens the properties of the transforms of the analogous problems for the particular case of infinite rate of heat propagation $c_p = \infty$, given in /1/. The results of /1/ were used in /7/ to derive a conclusion that the dynamic coupled problem of thermoelasticity does not satisfy the causality principle.

3. Generalized, centrally symmetric problem III⁻. Let us consider an infinite thermoelastic medium with a spherical cavity of radius r_0. The surface of the cavity is subjected to a mechanical and thermal action as follows:

$$\sigma_{rr} = -f_1(t)H(t),\quad \theta = u_4 = f_2(t)H(t),\quad r = r_0 \tag{3.1}$$

$$H(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

The resulting nonsteady fields of stresses, deformations and temperature satisfy the system (1.1), the boundary condition (3.1) and the condition of causality /6/. Using a complex Fourier transform, we write the solution in the form

$$\theta = \frac{\Phi}{r},\quad u_r = \frac{\partial}{\partial r}\left(\frac{\Psi}{r}\right) \tag{3.2}$$

$$\sigma_{rr} = (\lambda + 2\mu)\frac{\partial u_r}{\partial r} + 2\lambda\frac{u_r}{r} - (3\lambda + 2\mu)\alpha_\theta\theta$$

$$\Phi(r,t) = \frac{1}{\sqrt{2\pi}}\int_\Gamma\left[\sum_{p=1}^{2} C_p e^{i\lambda_p(r-r_0)}\right]e^{-i\omega t}\,d\omega \tag{3.3}$$

$$\Psi(r,t) = \frac{m_1}{\sqrt{2\pi}}\int_\Gamma\left[\sum_{p=1}^{2} C_p\left(\frac{\omega^2}{c_1^2} - \lambda_p^2\right)^{-1} e^{i\lambda_p(r-r_0)}\right]e^{-i\omega t}\,d\omega$$

$$C_p = (-1)^p\frac{1}{m_1}\left(\frac{\omega^2}{c_1^2} - \lambda_p^2\right)\left\{f_1^F(\omega)\frac{r_0^3}{\rho}\left(\frac{\omega^2}{c_1^2} - \lambda_{p\pm1}^2\right) - r_0 m_1 f_2^F(\omega)\times\right.$$
$$\left.[\omega^2 r_0^2 + 4c_2^2(ir_0\lambda_{p\pm1} - 1)]\right\}D^{-1},\quad p = 1, 2,$$

$$f_j^F(\omega) = \frac{1}{\sqrt{2\pi}}\int_0^\infty f_j(t)e^{i\omega t}\,dt,\quad j = 1, 2$$

$$D = (\lambda_2 - \lambda_1)\left[(\lambda_1 + \lambda_2)(\omega_2 r_0^2 - 4c_2^2) + 4ic_2^2 r_0\left(\lambda_1\lambda_2 + \frac{\omega_2}{c_1^2}\right)\right]$$

$$m_1 = \frac{\gamma}{\lambda + 2\mu},\quad c_2^2 = \frac{\lambda}{\rho}$$

The contour Γ in (3.3) is chosen in accordance with Sect.2.

Let us give the complete solution obtained by computing the integrals (3.3) with help of the properties of the characteristic parameters $\lambda_{1,2}$ of Lemma 1, for the case when the stress and temperature fields appear as the result of the action of thermal shock $f_2(t) = \theta_0 = \text{const}$, $f_1(t) = 0$ at the surface of the spherical cavity with $\mu = 0$, the case corresponding to a physical model of a thermoelastic liquid medium

$$\theta_0^{-1}\frac{R}{R_0}\theta(R,\tau) = H(\tau_+) + I_1(R,\tau)[H(\tau_-) - H(\tau_+)] + \Gamma_1(R,\tau) \tag{3.4}$$

$$(\theta_0\rho m_1 c_1^2)^{-1}\frac{R}{R_0}\sigma_{rr}(R,\tau) = I_2(R,\tau)[H(\tau_-) - H(\tau_+)] + \Gamma_2(R,\tau)$$

$$I_n(R,\tau) = \frac{1}{\pi}e^{-\chi_2^0\tau}\int_0^{\chi_1^0}(e^{-\alpha_+}\Psi_+ + e^{-\alpha_-}\Psi_-)\,dx,\quad n = 1, 2$$

$$\Psi_\pm = \frac{x\psi_\pm\delta_{1n}}{2dk_0^2}\sin\varphi_\pm - \frac{[(a-2)k_0^2 - \chi_2^0\psi_\pm]\delta_{1n} + \delta_{2n}}{d(2k_0^2\delta_{1n} + \delta_{2n})}\cos\varphi_\pm$$

$$\psi_\pm = 1 + \varepsilon \pm d,\quad d = \sqrt{p}\,[(\chi_1^0)^2 - x^2]^{1/2},\quad k_0 = [(\chi_2^0)^2 + x^2]^{1/2}$$

$$\varphi_\pm = (R - R_0)\gamma_\pm - \tau x,\quad \alpha_\pm = (R - R_0)\beta_\pm,\quad \tau_\pm = \tau - (R - R_0)c_\pm^{-1}$$

$$c_\pm = \sqrt{2}\,(a \pm \sqrt{p})^{-1/2}, \quad \gamma_\pm = 2^{-3/2}(\sqrt{Z^0 + X_\pm^0}K_+ + \sqrt{Z^0 - X_\pm^0}K_-)$$
$$Z^0 = [(X_\pm^0)^2 + (Y^0)^2]^{1/2}, \quad \beta_\pm = 2^{-3/2}(\sqrt{Z^0 + X_\pm^0}K_- - \sqrt{Z^0 - X_\pm^0}K_+)$$
$$K_\pm = (k_0 \pm \chi_2^0)^{1/2}, \quad X_\pm^0 = -\chi_2^0 a + 1 + \varepsilon \pm d, \quad Y^0 = -ax$$
$$\Gamma_n(R,\tau) = \begin{cases} J_n(0)\,H(\tau_-) + H(\tau_-) - H(\tau_+), & 0 \leqslant M^2 < (1-\varepsilon)(1+\varepsilon)^{-2} \\ J_n(\chi_2^0)\,[H(\tau_-) - H(\tau_+)] + J_n(0)\,H(\tau_+), & (1-\varepsilon)(1+\varepsilon)^{-2} < M^2 \leqslant (1-\varepsilon)^{-1} \\ J_n(0)\,H(\tau_+), & (1-\varepsilon)^{-1} \leqslant M^2 < \infty \end{cases}$$
$$J_n(b) = \frac{1}{\pi}\int_b^{M^{-2}} e^{-\tau x}\sin\left[(R - R_0)\sqrt{Ax}\right]\frac{\delta_{2n} - 2(A+x)\,\delta_{1n}}{h\,(2x\delta_{1n} + \delta_{2n})}\,dx, n=1,2$$
$$A = 2^{-1}(h + 1 + \varepsilon - ax), \quad h = \sqrt{p}\,[(\chi_2^0 - x)^2 + (\chi_1^0)^2]^{1/2}$$
$$\tau = \frac{c_1^2}{\varkappa}t, \quad R = \frac{c_1}{\varkappa}r, \quad R_0 = \frac{c_1}{\varkappa}r_0$$

The solution (3.4) implies that in the case of a generalized, coupled centrally symmetric problem of thermoelasticity under the mechanical and thermal action at the region's boundary, the resulting temperature and stress fields propagate with velocity c and undergo a jump at $\tau = (R - R_0)c_+^{-1}$. The solutions obtained hold for any value of the parameters R, τ, M, ε, while in /8/ the solution of a particular case of the problem with the inertial terms neglected was constructed using the method of perturbing the parameter ε and asymptotically for small values of time $\tau_\pm$. We also note that the solution (3.4) can be used to obtain the behavior of the stress and temperature fields when $\tau \to \infty$:

$$(\theta_0\rho m_1 c_1^2)^{-1}\frac{R}{R_0}\sigma_{rr}(R,\tau) = (R - R_0)\,\frac{\tau^{-3/2}}{2\sqrt{\pi(1+\varepsilon)}}, \qquad \theta_0^{-1}\frac{R}{R_0}\theta(R,\tau) = 1 - (R - R_0)\sqrt{\frac{1+\varepsilon}{\pi}}\,\tau^{-1/2}$$

which agrees with the conclusions of Sect.2.

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Translated by L.K.